How we teach calculations:

Calculation Protocol for Mathematics

Be the best that you can be!

About our calculation policy:

The following calculation policy has been written in response to the New National Curriculum 2014 and aims to give a consistent and smooth progression of learning, within mathematics, for pupils across the school. Please note that early learning in number and calculation in Reception follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations:

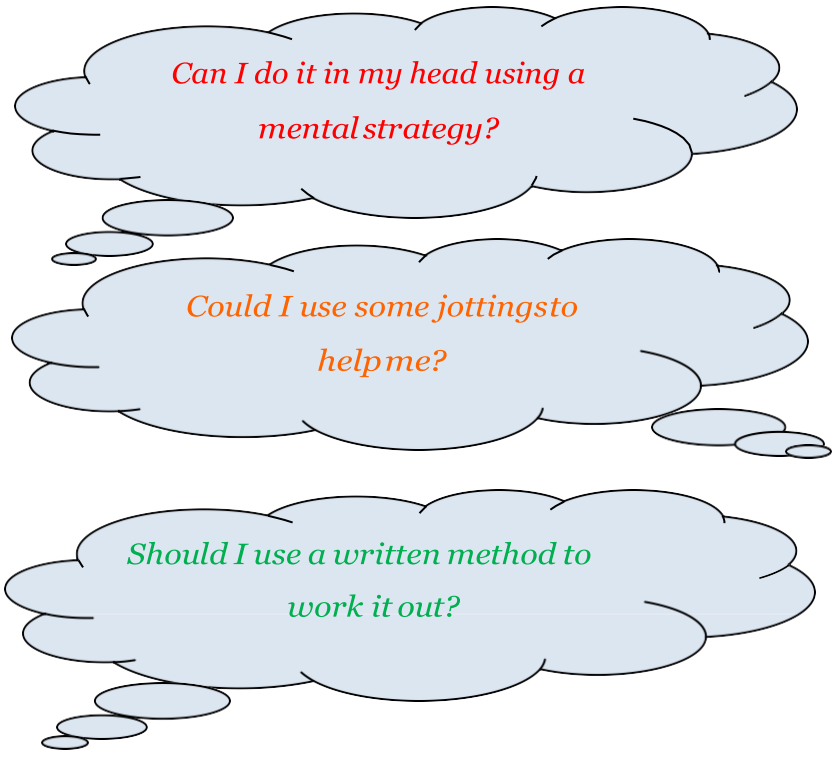
The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on – our aim is to create effective and efficient mathematicians.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons (one method does not meet all purposes).

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation; to ensure they select the most appropriate method for the numbers involved:



To work out a tricky calculation:
Approximate
Calculate
Check it

Rationale for KS1

Children in Years 1 and 2 will be given a really solid foundation in the basic building blocks of mental and written arithmetic. Through being taught place value, they will develop an understanding of how numbers work, so that they are confident in 2-digit numbers and beginning to read and say numbers above 100. A focus on number bonds, first via practical hands-on experiences and subsequently using memorisation techniques, enables a good grounding in these crucial facts, and ensures that all children leave Y2 knowing the pairs of numbers which make all the numbers up to 10 at least. They will also have experienced and been taught pairs to 20.

Their knowledge of number facts enables them to add several single-digit numbers, and to add/subtract a single digit number to/from a 2-digit number. Another important conceptual tool is their ability to add/subtract 1 or 10, and to understand which digit changes and why. This understanding is extended to enable children to add and subtract multiples of ten to and from any 2-digit number. The most important application of this knowledge is their ability to add or subtract any pair of 2- digit numbers by counting on or back in tens and ones. Children may extend this to adding by partitioning numbers into tens and ones.

Children will be taught to count in 2s, 3s, 5s and 10s, and will have related this skill to repeated addition. They will have met and begun to learn the associated 2x, 3x, 5x and 10x tables. Engaging in a practical way with the concept of repeated addition and the use of arrays enables children to develop a preliminary understanding of multiplication, and asking them to consider how many groups of a given number make a total will introduce them to the idea of division. They will also be taught to double and halve numbers, and will thus experience scaling up or down as a further aspect of multiplication and division. Fractions will be introduced as numbers and as operators, specifically in relation to halves, quarters and thirds.

Rationale for Lower KS2

In Years 3 and 4, children build on the concrete and conceptual understandings they have gained in Year 1 and 2 to develop a real mathematical understanding of the four operations, in particular developing arithmetical competence in relation to larger numbers.

In addition and subtraction, they are taught to use place value and number facts to add and subtract numbers mentally and will develop a range of strategies to enable them to discard the ‘counting in ones’ or fingers-based methods of the infants. In particular, they will learn to add and subtract multiples and near multiples of 10, 100 and 1000, and will become fluent in complementary addition as an accurate means of achieving fast and accurate answers to 3-digit subtractions. Standard written methods for adding larger numbers are taught, learned and consolidated, and written column subtraction is also introduced.

This key stage is also the period during which all the multiplication and division facts are thoroughly memorised, including all facts up to the 12 x 12 table. Efficient written methods for multiplying or dividing a 2-digit or 3-digit number by as single-digit number are taught, as are mental strategies for multiplication or division with large but friendly numbers, e.g. when dividing by 5 or multiplying by 20.

Children will develop their understanding of fractions, learning to reduce a fraction to its simplest form as well as finding non-unit fractions of amounts and quantities. The concept of a decimal number is introduced and children consolidate a firm understanding of one-place decimals, multiplying and dividing whole numbers by 10 and 100.

Rationale for UKS2

In Year 5 and 6, children move on from dealing mainly with whole numbers to performing arithmetic operations with both decimals and fractions. They will consolidate their use of written procedures in adding and subtracting whole numbers with up to 6 digits and also decimal numbers with up to two decimal places. Mental strategies for adding and subtracting increasingly large numbers will also be taught. These will draw upon children’s robust understanding of place value and knowledge of number facts. Efficient and flexible strategies for mental multiplication and division are taught and practiced, so that children can perform appropriate calculations even when the numbers are large, such as 40,000 x 6 or 40,000 ÷ 8.

In addition, it is in Y5 and Y6 that children extend their knowledge and confidence in using written algorithms for multiplication and division. Fractions and decimals are also added, subtracted, divided and multiplied, within the bounds of children’s understanding of these more complicated numbers, and they will also calculate simple percentages and ratios. Negative numbers will be added and subtracted. Algebra will also be introduced.

ADDITION

Year 1: Add with numbers up to 20

Use numbered number lines to add, by counting on in ones, encouraging children to begin with larger number and count on.

Children should:

- have access to a range of equipment eg. Number lines, counting apparatus, Numicon, 100, squares, bead strings etc
- be shown numbers in a range of contexts
- Read and write number sentences using the = and + signs
- Interpret number sentences including missing number problems e.g. 3 + = = 8

Key vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key Skills for addition at Year 1

- Read and write numbers to 100 in numerals (1-20 in words)
- Count to and across 100
- Recall bonds to 10 and 20, and addition facts within 20 ('story of' 5, 6, 7, 8, 9 and 10)
- Count on in ones from a given 2-digit number
- Add two single-digit numbers by counting on
- Add three single-digit numbers spotting doubles or pairs to 10
- Count on in tens from any given 2-digit number
- Add 10 to any given 2-digit number
- Use number facts to add single-digit numbers to two-digit numbers, e.g. use 4 + 3 to work out 24 + 3, 34 + 3...
- Add by putting the larger number first
- Recognise doubles to double 6

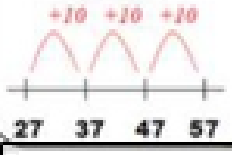
ADDITION

Year 2 Add with 2-digit numbers develop mental fluency with place value and addition using 2-digit numbers, then move to formal methods

Add 2-digit numbers and tens, 2-digits and units, two 2-digit numbers, first practically using equipment (Dienes Base 10, Numicon, 100squares) then using

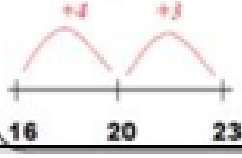
Add 2-digit numbers and tens:

27 + 30



Add 2-digit numbers and units:

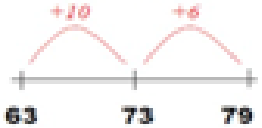
16 + 7



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:

63 + 16



23 + 34:

20	+	3
+ 30	+	4
50	+	7
	=	57

STEP 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 60+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

58 + 43:

50	+	8
+ 40	+	3
90	+	11
	=	101

STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key Skills for addition at Year 2

- Locate any 2-digit number on a landmarked line and use this to compare numbers; record comparisons < and >, e.g. 56 > 39.
- Identify any number on the 1-100 number grid; understand that each number is a multiple of ten and some ones, e.g. 54 is 50 and 4 more.
- Add two single digit numbers (8 + 7) by counting up; add two 2-digit numbers which total less than 100 by counting on in tens and ones, e.g. 54 + 37 as 54 + 30 + 7.
- Know securely number pairs for all the numbers up to and including 12
- Count in steps of 2, 5, and 10 from 0.
- Know different unit patterns when not crossing a ten, e.g. 4 + 3 = 7, 14 + 3 = 17, 24 + 3 = 27
- Begin to recognise unit patterns when crossing a ten, e.g. 5 + 6 = 11
- Know pairs with a total of 20 and multiples of 10 to 100
- Count on in ones and tens from any given 2-digit number
- Add two or three single-digit numbers
- Add a single-digit number to any 2-digit number using number facts, including bridging multiples of 10.
- Add 10and small multiples of 10 to any given 2-digit number
- Add any pair of 2-digit numbers
- Know that adding can be done in any order
- Solve problems with addition using concrete objects, pictorial representations, involving numbers, quantities and measures, applying written and mental methods

ADDITION

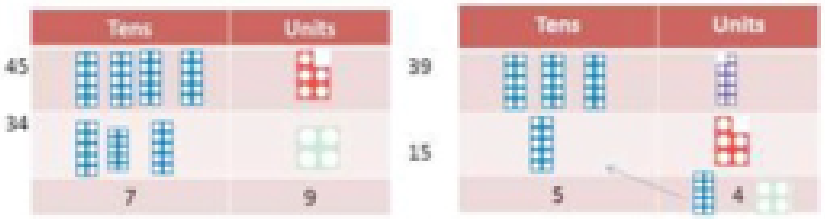
Year 3 Add numbers up to 3 digits

Use partitioning method for addition to add two or three 3-digit numbers or three 2-digit numbers (see year 2) Begin to use compact column addition to add numbers with three digits.

Use this intermediate step only if children experience difficulty moving on from partitioning method

When do we know children are ready for this method?

Do they know addition and subtraction facts to 20?
Do they understand place value and can they partition numbers?
Can they explain their mental strategies orally and record them using informal jottings?



236
+ 73
309
1

Add the units first, carry numbers underneath the bottom line, remind pupils of actual value e.g. 3 tens add 7 tens.

Children who are very secure and confident with 3-digit expanded column addition, should be moved onto the compact column addition method, involving carrying. A comparison of the partitioning addition method to compact method is useful to show minimising the number of steps

Key vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact

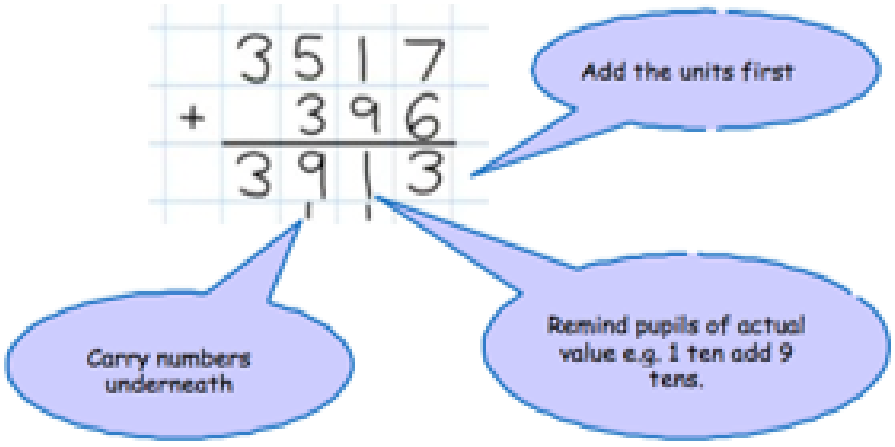
Key Skills for addition at Year 3

- Know pairs with each total to 20
- Know pairs of multiples of 10 with a total of 100
- Add any two 2-digit numbers by counting on in 10s and 1s or by using partitioning
- Add multiples and near multiples of 10 and 100
- Add 1,10, 100 to 3-digit numbers
- Understand place value in 3-digit numbers
- Perform place value additions without a struggle. (E.g. 300 + 8 + 50 = 358)
- Use place value and number facts to add a 1-digit or 2-digit number to a 3-digit number. (E.g. 104 + 56 is 160 since 104+50=154 and 6+4=10 and 676 + 8 is 684 since 8=4+4 and 76+4+4=84')
- Add pairs of 'friendly' 3-digit numbers mentally, e.g. 320 + 450
- Begin to add amounts of money using partitioning.
- Solve problems with addition using number facts, place value, missing numbers.

ADDITION

Year 4 Add numbers with up to 4 digits

Continue to use the compact column method, adding units first and carrying underneath he calculation. Also include money and measures contexts.



Children use and apply this method to money and measures.

Key vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, in- crease, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse

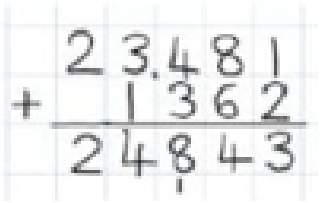
Key Skills for addition at Year 4

- Select appropriate method, mental, jottings, written—and explain why
- Add any two 2-digit numbers by partitioning or counting on
- Know by heart/quickly derive number bonds to 100 (e.g. 32 + 68) and to £1 (64p + 36p)
- Add to the next hundred, pound and whole number (e.g. 234 + 66 = 300, 3.4 + 0.6 = 4)
- Perform place value additions without a struggle. (E.g. 300 + 8 + 50 + 4000 = 4358)
- Add multiples and near multiples of 10, 100 and 1000.
- Add £1, 10p, 1p to amounts of money
- Use place value and number facts to add 1-, 2-, 3-and 4-digit numbers where a mental calculation is appropriate'. (E.g. 4004 + 156 by knowing that 6+4=10 and that 4004+150= 4154 so total is 4160)
- Perform inverse operations to check
- Solve 2-step problems in context
- Continue to practise a wide range of mental addition strategies e.g. Round and adjust, near doubles, numbers bonds, partitioning and recombining

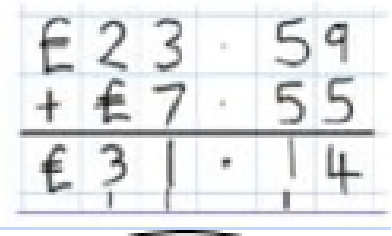
ADDITION

Year 5 Add numbers with more than 4 digits

Including money, measure and decimals with different numbers of decimal places

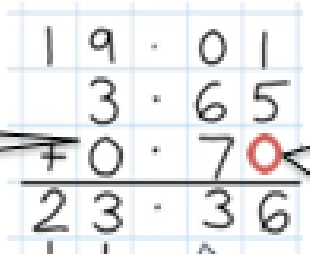


Use column addition to add two or three whole numbers.



Use column addition to add any pair of two-place decimal numbers including amounts of money.

Say 6 tenths and 7 tenths to reinforce place value



Empty decimal places can be filled to with zero to show the place value of each column

Children should:

Understand the place value of tenths and hundredths and use this to align numbers with differing numbers of decimal place.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, in- verse, decimal places, decimal point, tenths, hundredths, thousandths.

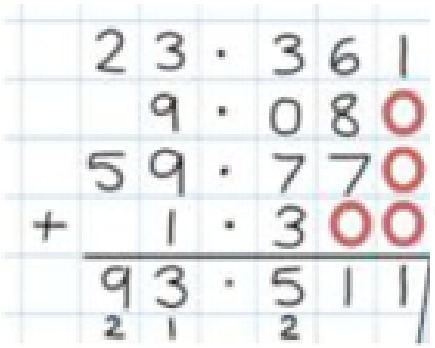
Key Skills for addition at Year 5

- Locate 5 and 6 digit numbers on a landmarked line; use this to compare/order numbers.
- Round to ten, a hundred, a thousand or ten thousand.
- Use rounding to check accuracy
- Understand a one-place decimal number as a number of tenths and a two-place decimal number as a number of hundredths.
- Add or subtract 0.1 or 0.01 to/from any decimal number with confidence, e.g. 5.83 + 0.01 or 4.83 – 0.1
- Add and subtract mentally with confidence – where the numbers are less than 100 or the calculation relies upon simple addition and place value.
- Confidently add numbers with more than 4-digits using a secure written method, including adding 'piles' of numbers
- Use inverse to check calculations

ADDITION

Year 6 Add several numbers of increasing complexity

Including money, measure and decimals with different numbers of decimal places

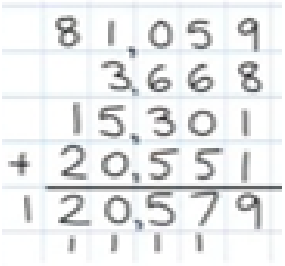


Tenths, hundredths and thousandths should be correctly aligned, with the decimal point aligned vertically, including in the answer.

Empty decimal places can be filled to with zero to show the place value of each column.

Use compact column method to add in context of money, measures, including decimals with different numbers of decimal places.

Pupils should apply their knowledge of a range of mental strategies, mental recall skills, informal and formal written methods when selecting the **appropriate method** to work out addition problems. Opportunities to discuss the appropriateness of methods need to be planned for.



Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, carry, expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths*

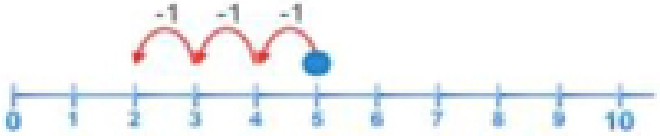
Key skills for addition at Year 6:

- Perform mental calculations, including with mixed operations and large numbers, using and practicing a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

SUBTRACTION

Year 1 Subtract from numbers up to 20

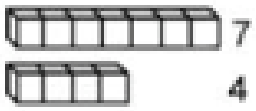
Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below (and then empty ones):



5 - 3 = 2

Model subtraction practically and using number tracks, number lines and 100 squares and practically. Find the difference between - this is to be done practically using the language 'find the distance between' and 'how many more than?

This will be introduced practically with the language 'find the distance between' and 'how many more?' in a range of familiar contexts.



'7 is 3 more than 4.'

'I am 3 years older than my sister.'

Key vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...

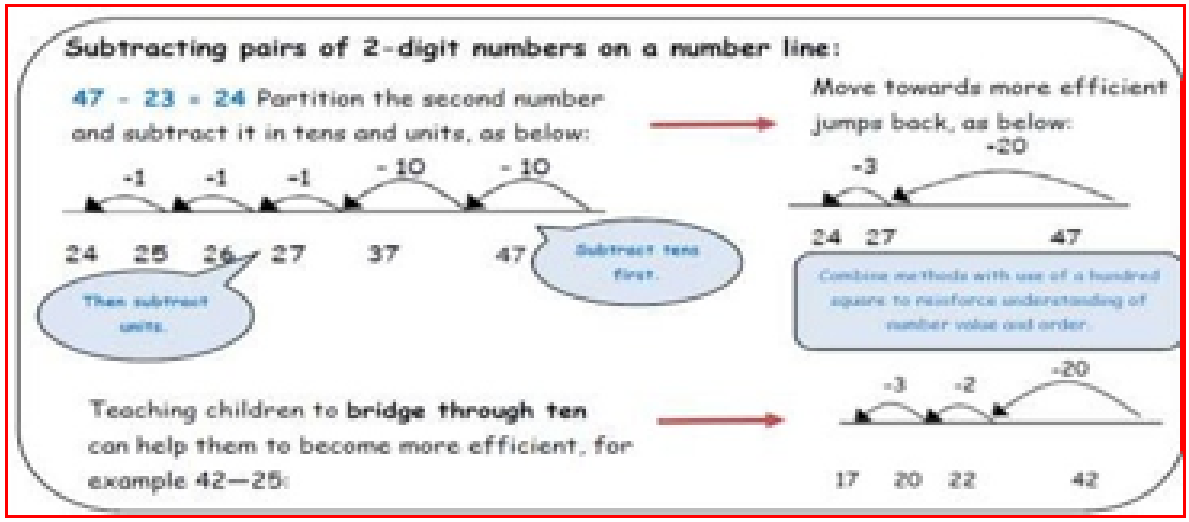
Key Skills for subtraction at Year 1

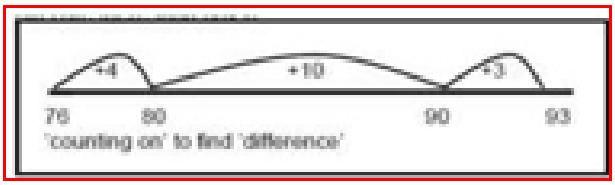
- Give a number, say one less
- Count back in ones to from 100 and from any single-digit or 2-digit number.
- Count back in tens from any 2-digit number
- Locate any number on a 1-100 grid or a beaded line 0-100.
- Know number bonds to 10, also know what is left if objects are taken from 10, e.g. 10 fingers, fold down 4; leaves 6 standing.
- Solve one-step problems involving subtraction, using concrete objects (bead strings, objects, cubes) and pictures, and missing number problems
- Recognise the - and = signs, and use these to read and write simple subtractions.

SUBTRACTION

Year 2 Subtract with 2-digit numbers Use practical equipment such as Dienes and Numicon to model subtraction.

Subtract first on a numbered number line, then on an empty number line, by counting back, aiming to develop mental subtraction skills.





Children should also learn how to count on in order to find the difference. They should be given opportunities to explore when to count on and when to count back.

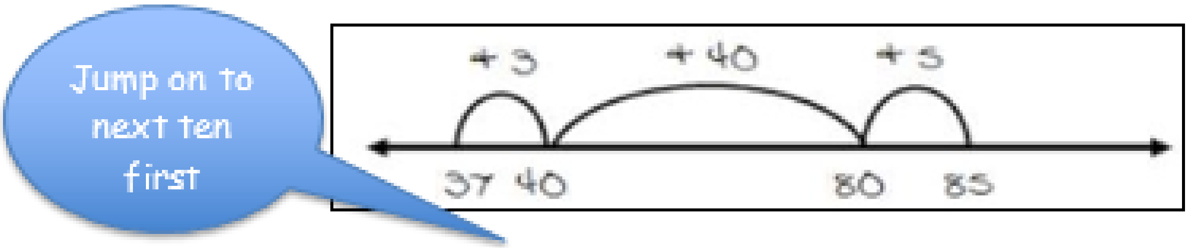
Key vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units

Key Skills for subtraction at Year 2

- Recognise that addition and subtraction are inverse operations and understand that 10 – 4 = 6 as well as 6 + 4 = 10.
- Count back in ones or tens to take away, e.g. 27 – 3 = or 54 – 20 =.
- Begin to count up to find a difference between two numbers with a small gap (42 – 38). Know when to count on and when to count back
- Recall and use subtraction facts to 20 fluently
- And derive and use related fact to 100
- Subtract using concrete objects, pictorial representations, 100 squares, Dienes, Numicon and mentally, including a 2-digit number and ones, a 2-digit numbers and tens, and two 2-digit numbers
- Use inverse to check calculations.

SUBTRACTION

Year 3 Subtract with 2 and 3-digit numbers – subtract on an empty number line (ENL) by counting on.



Children should understand when to count back where appropriate, using place value or number facts. This skill should be reinforced through mental work.

Begin to use formal column subtraction method, first using 'friendly numbers'.

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 38
-12
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 26
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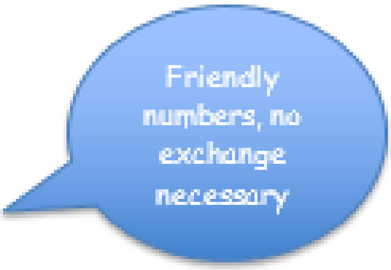
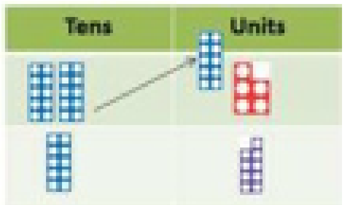


Move to formal subtraction using 'take and make'.

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 5
 6̸ ¹3
-1 5
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 4 8
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Teach the children to consider the most appropriate method.

Key vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds

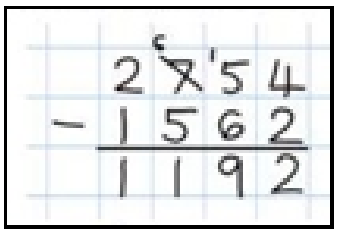
Key Skills for subtraction at Year 3

- Understand place value in 3-digit numbers; add and subtract 1s, 10s or 100s without difficulty; use this to add and subtract multiples of 1, 10, 100 to/from 3-digit numbers.
- Mentally subtract any pair of 2 digit numbers, e.g. 75 - 58
- Recognise that there are two ways of completing subtractions, either by counting up (using ENL) or by counting back, e.g. 54 - 3 (counting up)
- Subtract mentally using place value and number bonds, eg. 347-5, 347-40, 347-100)

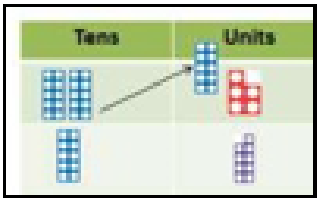
SUBTRACTION

Year 4 Subtract with up to 4-digit numbers

Subtract using formal column subtraction, using take and make where appropriate.

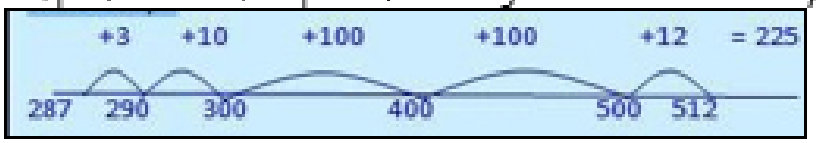


Use Numicon and Dienes to provide visual image for 'take and make'



Use complementary addition to subtract amounts of money, and for subtractions where the larger number is a near multiple of 1000 or 100

Use counting up with confidence to solve most subtractions, including finding complements to multiples of 100. E.g. 512 - 287 is done by:



NB. Children should be encouraged to progress to using the fewest number of jumps

Key vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse

Key Skills for subtraction at Year 4

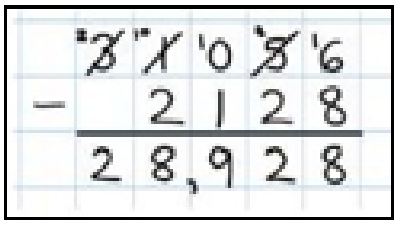
- Mentally subtract any pair of two digit numbers.
- Subtract 3 digit numbers from 3 digit numbers using counting on, e.g. 426 - 278 by jumping along a line from 278 to 426
- Practise mental subtraction strategies, e.g. Round and adjust (37—9), using place value
- Use counting on in the context of money and also when subtracting from numbers ending in zeros e.g. 4000-372
- Count backwards through zero, using negative numbers

SUBTRACTION

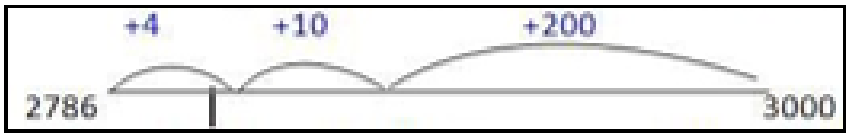
Year 5 Subtract with at least 4-digit numbers

Including money measures and decimals.

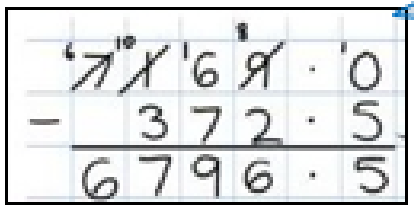
Use compact column subtraction to subtract numbers with up to 5 digits.



Use counting on for subtractions where the larger number is a multiple or near multiple of 1000, or for decimals.



Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point.



Add a zero in any empty decimal place to aid understanding of what to subtract.

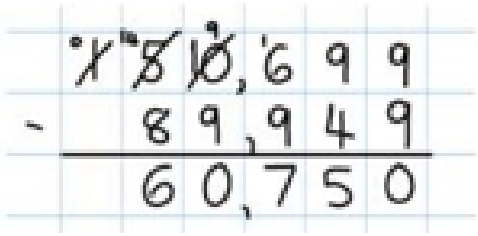
Key vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal

Key Skills for subtraction at Year 5

- Count backwards through zero, using negative numbers
- Add or subtract 0.1 or 0.01 to/from any decimal number with confidence, e.g. 5.83 + 0.01 or 4.83 – 0.1
- Children need to utilise and consider a range of subtraction strategies, jottings and written methods before choosing how to calculate
- Subtract larger numbers using column subtraction or by counting up
- Begin to subtract decimal numbers using counting up: 6.2 – 3.5
- Decide which mental methods to use and explain why

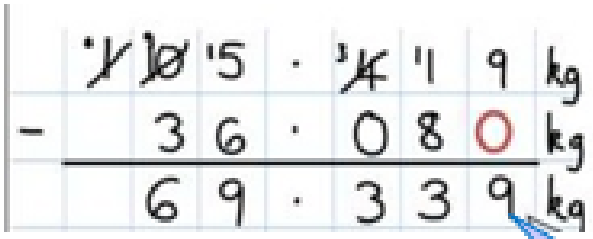
SUBTRACTION

Year 6 Subtracting with increasingly large and more complex numbers and decimal values.



Including money, measure and decimals with different numbers of decimal places

Use the compact column method to subtract more complex integers



Use compact column method to subtract in context of money, measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled to with zero to show the place value of each column.

Pupils should apply their knowledge of a range of mental strategies, mental recall skills, informal and formal written methods when selecting the **appropriate method** to work out subtraction problems. Opportunities to discuss the appropriateness of methods need to be planned for.

Key Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is..., difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal

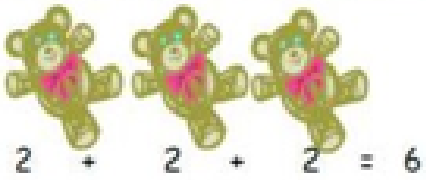
Key Skills for subtraction at Year 6

- Subtract mentally with confidence - where the numbers are less than 100 or the calculation relies upon simple subtraction and place value. Examples include: 6,723 - 400, 72 - 46, 100 - 64
- Subtract large numbers using column subtraction or counting up, e.g. 1323 - 758
- Subtract decimal numbers using counting up
- Use negative numbers in context and calculate intervals across zero
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before deciding how to calculate
- Decide which methods to use and explain why

MULTIPLICATION

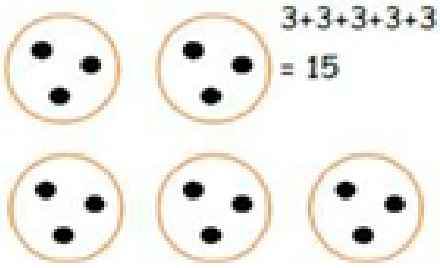
Year 1 Multiply with concrete objects, arrays and pictorial representations.

How many legs will 3 teddies have?



2 + 2 + 2 = 6

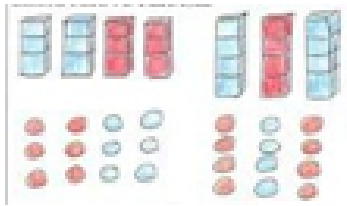
There are 3 sweets in one bag. How many sweets are in 5 bags altogether?

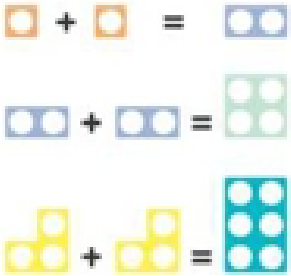


3+3+3+3+3 = 15

Count in 2s, 5s, 10s

Use visual and concrete arrays and 'sets of' objects to find the answers to '3 lots of 4', '2 lots of 5' etc.





Use Numicon to find doubles to double 6

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

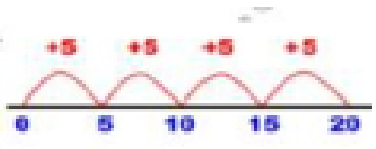
Key Skills for multiplication at Year 1

- Count in multiples of 2, 5 and 10
- Recognise doubles to double 6
- Solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

MULTIPLICATION

Year 2 Multiplication using arrays and repeated addition.

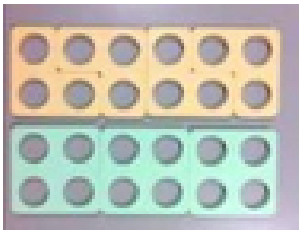
(Using at least 2s, 5s and 10s)



Starting from zero, make equal jumps on a number line to work out multiplication facts and write multiplication.

4 X 5 = 20

Use repeated addition on a number line.



Use arrays and Numicon to help teach children to understand the commutative law of multiplication.

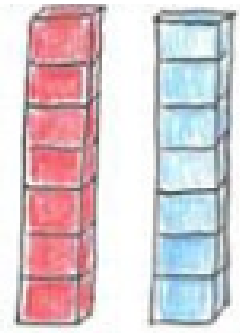
5 x 3 = 15

3 x 5 = 15

Learn doubles to double 20

Begin to double multiples of 5 to 100

Begin to double two-digit numbers less than 50 with 1s digits of 1, 2, 3 4 or 5



Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…

Key Skills for multiplication at Year 2

- Count in steps of 2, 3 and 5 from zero and in 10s from any number
- Know the 2X, 5X and 10X tables and begin to say how many 10s are in 40 or how many 5s are in 30; recognise odd and even answers
- Write and calculate number statements using x and = signs
- Show that multiplication can be done in any order
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, Numicon, mental methods and multiplication facts

MULTIPLICATION

Year 3 multiply 2-digit numbers by a single digit number

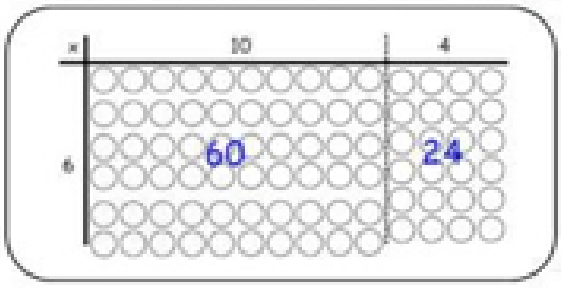
Introduce the grid method for multiplying 2 digits by 1 digit

Eg. 23 x 8 = 184

X	20	3
8	160	24

160 + 24 = 184

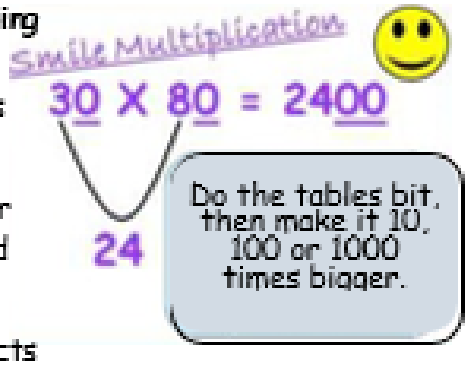
Link the layout of the grid to an array initially:



Demonstrate how the array links to the grid calculation

Children MUST be able to do the following before moving onto grid method:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (Smile multiplication) using their knowledge of multiplication facts and times tables.
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.



Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value

Key Skills for multiplication at Year 3

- Understand that multiplication is commutative, e.g. 4 x 8 is the same as 8 x 4.
- Know the 2x, 3x, 5x and 10x times tables. All tables need to be learned to 12th multiple.
- Multiply any 2-digit number by 10 or a single-digit number by 100;
- Understand the effect of multiplying whole numbers by 10 and 100.
- Multiply a 1-digit number by a 2-digit number starting to use the grid
- Solve multiplication problems involving missing numbers

MULTIPLICATION

Year 4 Multiply 2 and 3 digits by a single digit

using all multiplication tables up to 12 x 12

Developing the grid method:

Eg. 136 x 5 = 680

X	100	30	6
5	500	150	30

500
150
+ 30
680

Encourage mental addition or use of column addition to add accurately.

Move onto short multiplication (see Y5) if and when children are confident and accurate multiplying 2 and 3 digit numbers by a single digit this way and are already confident in carrying for written addition.

Children should be able to:

- Approximate before they calculate and make this a regular part of their calculating, going back to their approximation to consider the reasonableness of their answer.
- Record an approximation to check their answer against.
- Multiply multiples of 10 and 100 by a single digit, using smile multiplication.
- Recall all times tables up to 12 x 12

Smile Multiplication

30 X 80 = 2400

24

Do the tables bit, then make it 10, 100 or 1000 times bigger

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value, inverse

Key Skills for multiplication at Year 4

- Multiply 1 and 2 digit numbers by 10, 100 and 1000; to understand place value in decimal numbers with one place.
- Know and recite 2x, 3x, 4x, 5x, 9x, 10x times tables up to 12th multiple; include multi- plying by 0 (e.g. 5 x 0 = 0, 7 x 0 = 0) or by 1 (e.g. 5 x 1 = 5, $\frac{1}{2}$ x 1 = $\frac{1}{2}$).
- Multiply 1- digit numbers by 2-digit or friendly 3-digit numbers using grid method.
- Find doubles to double 100 and beyond, using partitioning
- Begin to double amounts of money
- Use doubling as strategy for multiplying by 2, 4, 8
- Count in multiples of 6, 7, 9, 25 and 1000

MULTIPLICATION

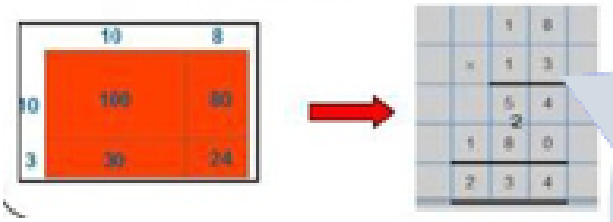
Year 5 Multiply up to 4 digits by 1 or 2 digits.

Introducing column multiplication

Introduce column multiplication by comparing a grid method calculation, in order to see how the steps are related. Notice how there are less steps involved.

x	300	20	7
4	1200	80	28

Introduce long multiplication for multiplying by 2 digits

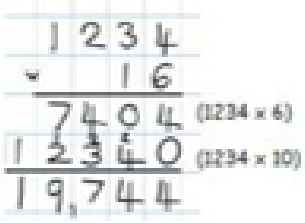


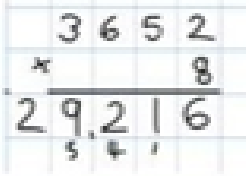
18 x 3 on the first row

(8 x 3 =24, carrying the 2 for 20, then 1 x 3)

18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first

Move towards more complex numbers





Children should approximate first

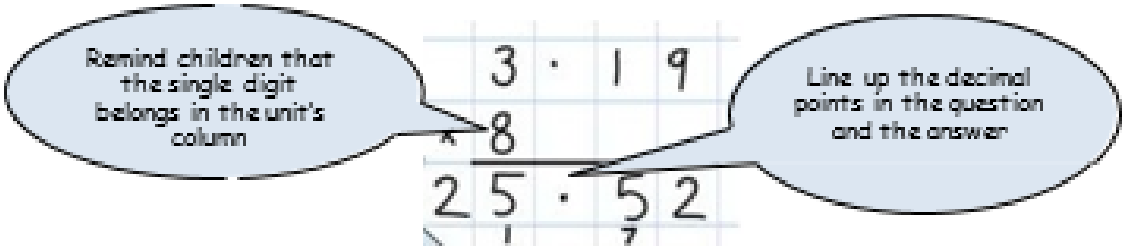
Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'

Key Skills for multiplication at Year 5

- Know and recite all times tables including division facts.
- Multiply 2- and 3-digit numbers by numbers ≤12 using grid method; multiply 2-digit by 2-digit numbers using grid method.
- Identify multiples and factors, using knowledge of multiplication tables up to 12 x 12
- Scale up or down by a factor of 2, 5 or 10
- Multiply integers and decimals by 10, 100, 1000
- Recognise and use squared, cubes and their notations

MULTIPLICATION

Year 6 Short and long multiplication, as in year 5, and multiply decimals with up to 2 decimal places by a single digit.



Use this method for money and measures.

Children should:

- Use rounding and place value to make approximations before calculating and use these to check validity of answers
- Use short multiplication to (see Y5) to multiply numbers with more than 4 digits by a single digit; to multiply money and measures; and to multiply decimals up to 2 decimal places by a single digit
- Use long multiplication (see Y5) to multiply numbers with at least 4 digits by a 2-digit number

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry', tenths, hundredths, decimal

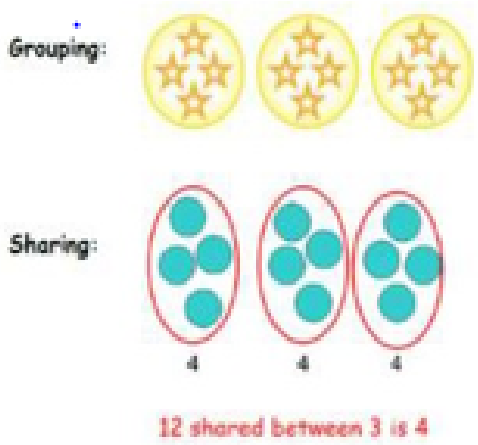
Key Skills for multiplication at Year 6

- Recall multiplication facts up to 12 x 12
- Use short multiplication to multiply a 1-digit number by a number with up to 4 digits
- Use long multiplication to multiply a 2-digit by a number with up to 4 digits
- Use short multiplication to multiply a 1-digit number by a number with one or two decimal places, including amounts of money.
- Multiply fractions and mixed numbers by whole numbers.
- Multiply fractions by proper fractions.
- Use percentages for comparison and calculate simple percentages.
- Estimate answers using rounding and approximation.

DIVISION

Year 1 Group and share small quantities

Using both objects diagrams and pictorial representations, to solve problems involving both grouping and sharing.



Children should solve a division problem within a context.

E.g. 5 children share 15 sweets. How many does each child get?

Can they solve this and write a division statement e.g. 15 sweets shared between 5 children gives 3 each.

Pupils should:

- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find half of a group of objects by sharing into 2 equal groups.

Key vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array

Key Skills for division at Year 1

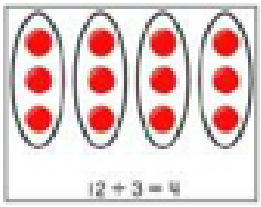
- Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

DIVISION

Year 2 Group and share using the ÷ and = signs.

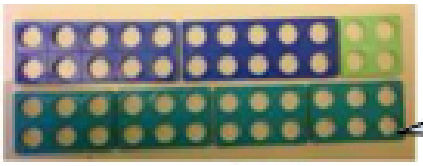
Use objects, Numicon, arrays, pictorial representations and grouping

Arrays:



This represents 12 ÷ 3, posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally.

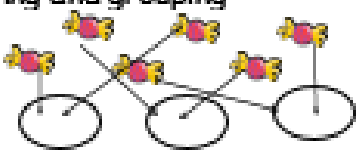


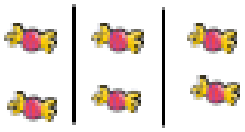
24 divided into groups (chunks) of 6

There are 4 groups of 6 in 24

Know and understand sharing and grouping

6 sweets shared between 3 people, how many do each get?

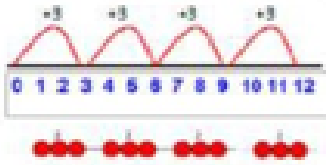




Grouping using a number line

Group from zero in equal jumps to find 'how many groups of _ in _?

Use bead-bars/strings to make link to number line.



Pose 12÷3 as "How many groups of 3 are there in 12?"

12 ÷ 3 = 4

Key vocabulary share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

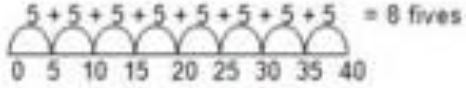
Key Skills for division at Year 2

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

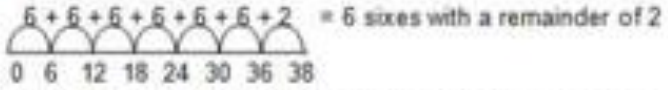
DIVISION

Year 3 Divide 2-digit numbers by a single digit

Example without remainder:
40 ÷ 5
Ask 'How many 5s in 40?'

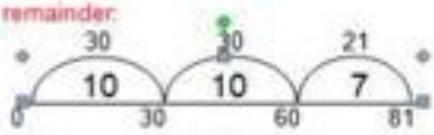


Example with remainder:
38 ÷ 6



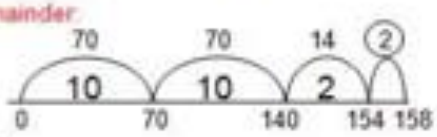
For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

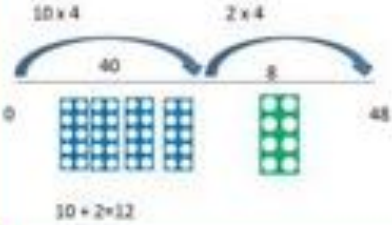
Example without remainder:
81 ÷ 3



This could either be done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That's 27 threes altogether) or by counting in jumps of known multiples of 3 to reach 81 (30 + 30 + 21) then working out the number of threes in each jump.

Example with remainder:
158 ÷ 7





Grouping on a number line first without, then with remainders

Model first using Dienes, then using bead bar to show link to ENL

Key vocabulary share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry ', remainder, multiple

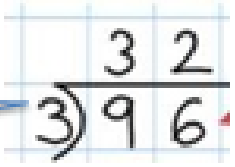
Key Skills for division at Year 3

- Recall and use division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one digit
- Solve problems, in contexts, and including missing number problems, involving division.
- Pupils develop efficient mental methods, for example, using division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2- digit numbers by 1-digit numbers using an ENL.
- Halve even numbers up to 50 and multiples of ten to 100
- Perform divisions within the tables including those with remainders, e.g. 38 ÷ 5.

DIVISION

Year 4 Divide up to 3-digit numbers by a single digit.

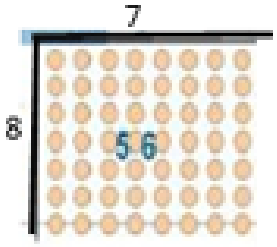
Short division: limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the devisor)



Remind children of correct place value (96 is equal to 90 and 6).

Once children are secure with division as grouping, demonstrate this using number lines, arrays etc. short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all.

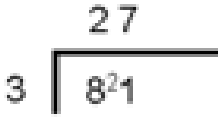
Start by introducing the layout of short division by comparing it to an array.



Example without remainder:

81 ÷ 3

81 is partitioned into multiples of 3 which are then each divided by 3	Over time, this is refined so that the dividend is partitioned into the highest multiple of the divisor that is also a multiple of 10, plus any remaining ones
10 + 10 + 7 3 | 30 + 30 + 21	20 + 7 3 | 60 + 21



Possibly use Numicon to model this expanded division

Move to short division with remainders (modeling process as expanded division first as seen above).

Example with remainder:

Key vocabulary share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, in- verse, short division, 'carry ", remainder, multiple, divisible by, factor

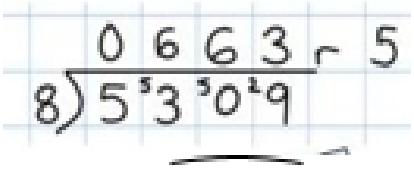
Key Skills for division at Year 4

- Use a written method to divide a 2-digit or a 3-digit number by a single-digit number.
- Give remainders as whole numbers.
- Recall multiplication and division facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

DIVISION

Year 5 Divide up to 4 digits by a single digit < or = to 12, including answers with remainders

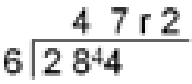
Short division including remainder answers. Please refer to Y4 or Y3 if necessary to ensure children are confident in the steps towards short division.



The answer could be expressed as 663 remainder 5 or 663 and 5/8 or as a decimal.

Division should be given in a real life context, including using money and measures, so that pupils know to round the answer up or down.

Answers could also be given as remainders, decimals or fractions.



Once children's understanding of this method is secure they might shorten their dialogue to:

"How many 6s in 28?"
"4 remainder 4"
"How many 6s in 44?"
"7 remainder 2"

BUT ensure children have a secure understanding of what they are doing and are able to use their knowledge of related facts to either make a rough estimate first or have an idea about whether their final answer is reasonable or not.

Key vocabulary share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry ', remainder, multiple, divisible by, factor quotient, prime number, prime factors, composite number (non-prime)

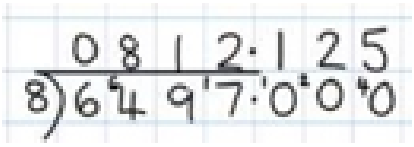
Key Skills for division at Year 5

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses. Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 24½ = 24.5 ≈ 25).

DIVISION

Year 6 Divide at least 4-digit numbers by single and 2-digit numbers (including decimals).

Short division (for dividing by a single digit)



Children should continue to use short division with remainders. They need to learn how to express an answer as a remainder, a fraction or as a decimal as in in this example.

It is important for children to start from real life problem solving contexts and for them to consider how best to approach a problem.

Introduce long division by chunking for dividing by 2 digits

Useful list:

1x = 36

10x = 360

100x = 3600

```
      27
36 ) 972
   - 720   (20x)
     252
   - 252   (7x)
       0
Answer : 27
```

Find out 'How many 36s are in 972?' by subtracting 'chunks' of 36, until zero is reached (or until there is a remainder).

Teach pupils to write a 'useful list' first at the side that will help them decide what chunks to use.

Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots? Can use 100 lots?' As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their 'useful' lists.

Teachers must consult division progression methods of previous years in order to determine valid starting points for children in year 6.

Key vocabulary as previously & common factor

Key Skills for division at Year 6

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.